Problem generalization Feuerbach–Luchterhand's theorem

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Abstract

Feuerbach–Luchterhand's theorem states that: Let ABCD be cyclic quadrilateral, D be a point on the plane then:

$$PA^2.DB.BC.CD - PB^2.AC.CD.DA + PC^2.BD.DA.AB - PD^2.CA.AB.BC = 0 \quad (1)$$

I proposed problem generalization Feuerbach–Luchterhand's theorem following(**Note: on classical geometry**):

Problem 1. Let convex cyclic polygons $A_1A_2A_3...A_{2n}$, and P be a point on the plane. Define:

$$T_k = \frac{A_{k-1}A_{k+1}}{A_{k-1}A_k.A_kA_{k+1}}.A_1A_2.A_2A_3....A_{2n}A_1 \quad (2)$$

Show that:

$$\sum_{k=1}^{2n}(-1)^{k+1}.PA_k^2.T_k = 0 \quad (3)$$

Problem 2 (Generalization problem 1)**.** Let two similar convex cyclic polygons $A_1A_2A_3...A_{2n}$ and $B_1B_2B_3...B_{2n}$ on the plane. Define:

$$T_k = \frac{A_{k-1}A_{k+1}}{A_{k-1}A_k.A_kA_{k+1}}.A_1A_2.A_2A_3....A_{2n}A_1 \quad (4)$$

Show that:

$$\sum_{k=1}^{2n}(-1)^{k+1}.B_kA_k^2.T_k = 0 \quad (5)$$

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