

A conjecture related with the Fermat Last Theorem

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Abstract

I propose a conjecture of generalization of the Fermat last theorem and the Lander, Parkin, and Selfridge conjecture.

Conjecture 1. Let n, m be two positive integers and $m \neq n$, let a be a integer and $a \neq 0$. Let $g(x)$ be a given polynomial, let $f(x) = g(x) + a.x^k$, then exist k_0 be the positive integer such that when $k \geq k_0$ then no $(n + m)$ positive integers $x_1, x_2, x_n, y_1, y_2, \dots, y_m$ greater than 1 can satisfy an equation as follows:

$$f(x_1) + f(x_2) + \dots + f(x_n) = f(y_1) + f(y_2) + \dots + f(y_m) \quad (1)$$

Example 1: Let $g(x) \equiv 0$, exist k_0 such that when $k \geq k_0$ then no three positive integers x, y, z can satisfy an equations:

$$x^k + y^k = z^k$$

By the Fermat Last Theorem, in **example 1** we get $k_0 = 3$

Example 2: Let n, m be two positive integers and $m \neq n$, let $g(x) \equiv 0$, exist k_0 such that when $k \geq k_0$ then no $(n + m)$ positive integers $x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_m$ can satisfy an equation:

$$x_1^k + x_2^k + \dots + x_n^k = y_1^k + y_2^k + \dots + y_m^k$$

By the Lander, Parkin, and Selfridge conjecture in **example 2**, $k_0 = n + m + 1$

Example 3: Let $g(x) = 4x^2 + 5$, exist k_0 such that when $k \geq k_0$ then no three integers x, y, z can satisfy an equation:

$$3x^k + 4x^2 + 5 + 3y^k + 4y^2 + 5 = 3z^k + 4z^2 + 5$$

Example 4: Let $g(x) = x$, exist k_0 such that when $k \geq k_0$ then no three integers x, y, z can satisfy an equation:

$$x^k + x + y^k + y = z^k + z$$

References

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