Another Generalization of the Sawayama's Lemma and Sawayama and Thébault's Theorem

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Abstract

1 A generalization of the Sawayama lemma

Theorem 1 (Case 1)**.** *Let ABC be a triangle, Let I be the incenter of ABC. Let (O) be a circle through B, C such that A outside of (O). Let (O_A) be the circle such that (O_A) tangent to AB, (O_A) tangent to AC, and externally tangent to (O). Le P be a point in the plane, let L be a line through P and tangent to (O_A). Let (O_1) be the circle such that (O_1) tangent with BC at D, (O_1) tangent L at E, and (O_1) tangent (O), such that (O_1) and (O_A) are not the same half plane divides by L. Then show that D, E, I are collinear*

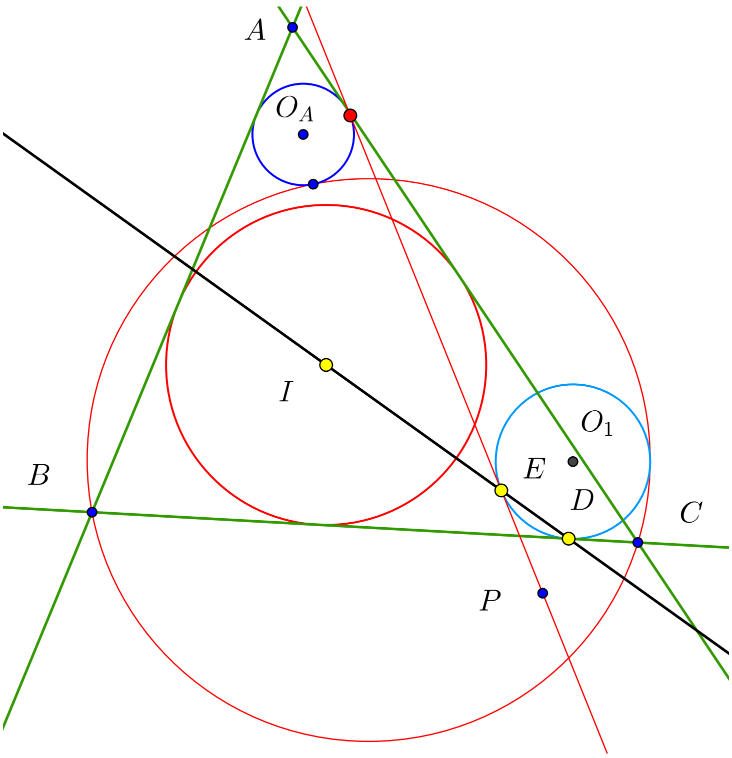

Figure 1

Theorem 2 (Case 2)**.** *Let ABC be a triangle, Let I be the incenter of ABC. Let (O) be a circle through B, C such that A inside of (O). Let (O_A) be the circle such that (O_A) tangent to AB, (O_A) tangent to AC, and tangent internaly with (O). Le P be a point in the plane, let L be a line through P and tangent to (O_A). Let (O_1) be the circle such that (O_1) tangent with BC at D, (O_1) tangent L at E, and (O_1) tangent (O), such that (O_1) and (O_A) are the same half plane divides by L. Then show that D, E, I are collinear.*

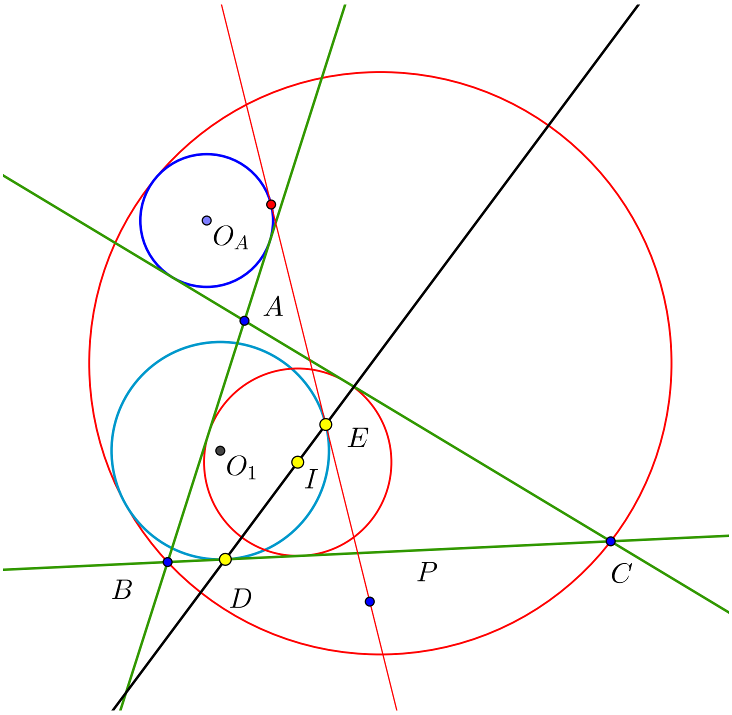

Figure 2

2 A generalization of the Sawayama-Thebault theorem

Theorem 3 (Case 1)**.** *Let ABC be a triangle. Let (O) be a circle through B, C such that A outside of (O). Let (O_A) be the circle such that (O_A) tangent to AB, (O_A) tangent to AC, and externally tangent to (O). Le P be a point in the plane, let L_1, L_2 be a line through P and tangent to (O_A). Let $(O_1), (O_2)$ be the circle such that (O_1), (O_2) tangent to L_1, L_2 respectively, and $(O_1), (O_2)$ tangent (O), $(O_1), (O_2)$ tangent to BC, such that (O_1) and (O_A) are not the same half plane divides by L_1. such that (O_2) and (O_A) are not the same half plane divides by L_2. The line O_1O_2 through a fixed point when P move on a line.*

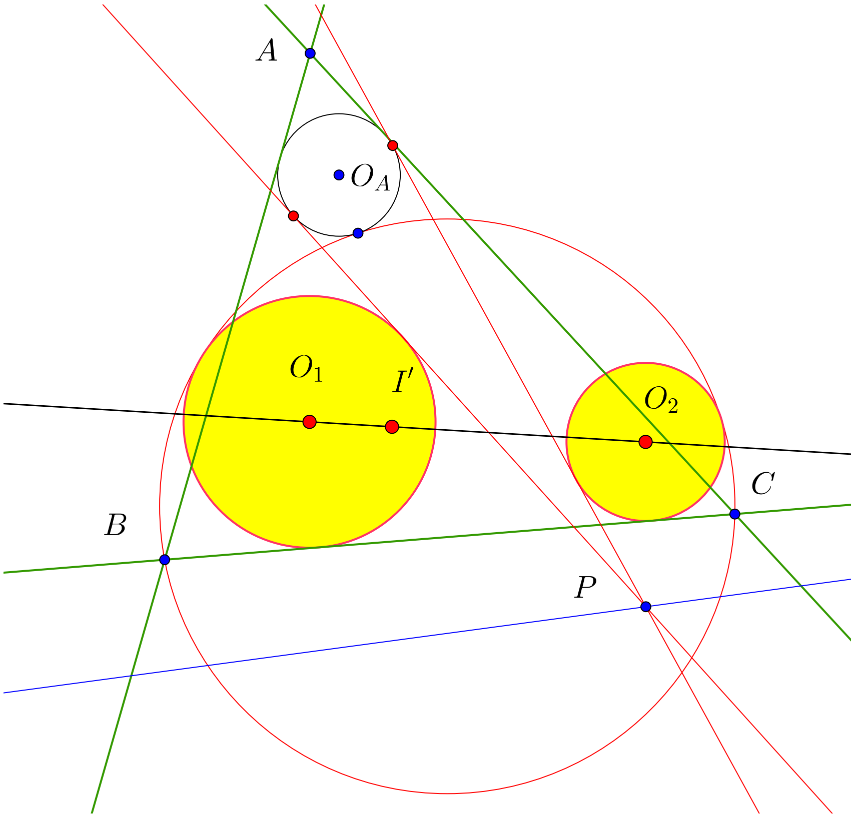

Figure 3

Theorem 4 (Case 2)**.** *Let ABC be a triangle. Let (O) be a circle through B, C such that A outside of (O). Let (O_A) be the circle such that (O_A) tangent to AB, (O_A) tangent to AC, and externally tangent to (O). Le P be a point in the plane, let L_1, L_2 be a line through P and tangent to (O_A). Let $(O_1), (O_2)$ be the circle such that (O_1), (O_2) tangent to L_1, L_2 respectively, and $(O_1), (O_2)$ tangent (O), $(O_1), (O_2)$ tangent to BC, such that (O_1) and (O_A) are the same half plane divides by L_1 and (O_2) and (O_A) are the same half plane divides by L_2. The line O_1O_2 through a fixed point when P move on a given line.*

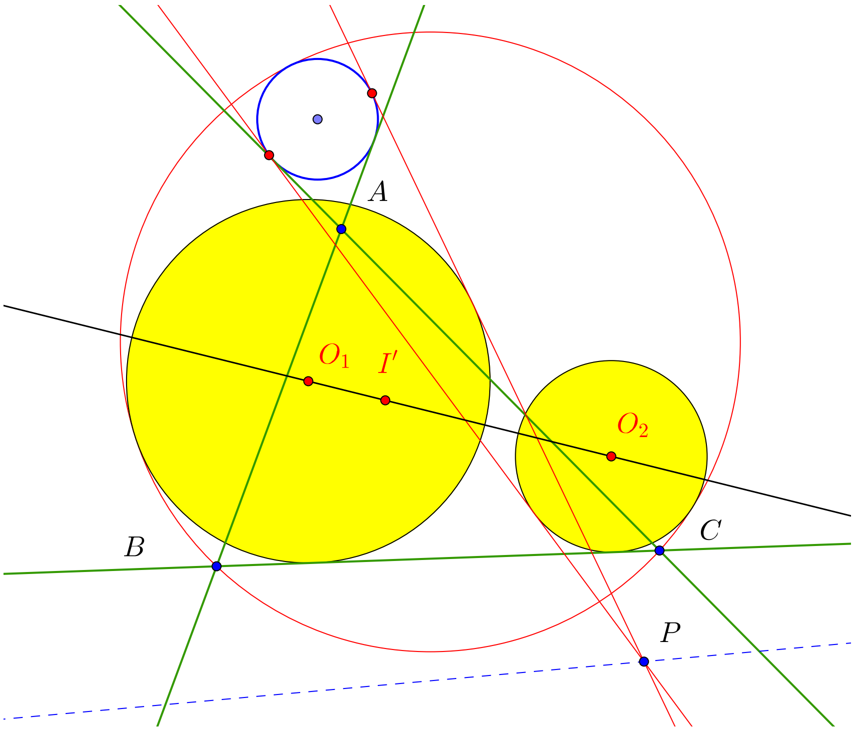

Figure 4

References

[1] Jean-Louis Ayme, *Sawayama and Thébault's Theorem*, Forum Geometricorum 3 (2003) 225–229.

[2] Dao Thanh Oai, *A Generalization of Sawayama and Thébault's Theorem*, International Journal of Computer Discovered Mathematics, Volume 1 Number 3 (September 2016) pp.33-35.

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