

Some Problems On Apollonian Gasket

Dao Thanh Oai

April 21, 2018

Abstract

I proposed some problems on Apollonian gasket configuration

Consider ABC be a triangle, construct three circles (A) , (B) , (C) such that they are tangent to each other. Let (A_1) be the circle tangent to the Soddy circle and (B) and (C) , let (A_{k+1}) be the circle tangent to (A_k) and (B) and (C) for $k = 2, 3, \dots, n$ define (B_i) , (C_i) cyclically. We have some problems in next pages.

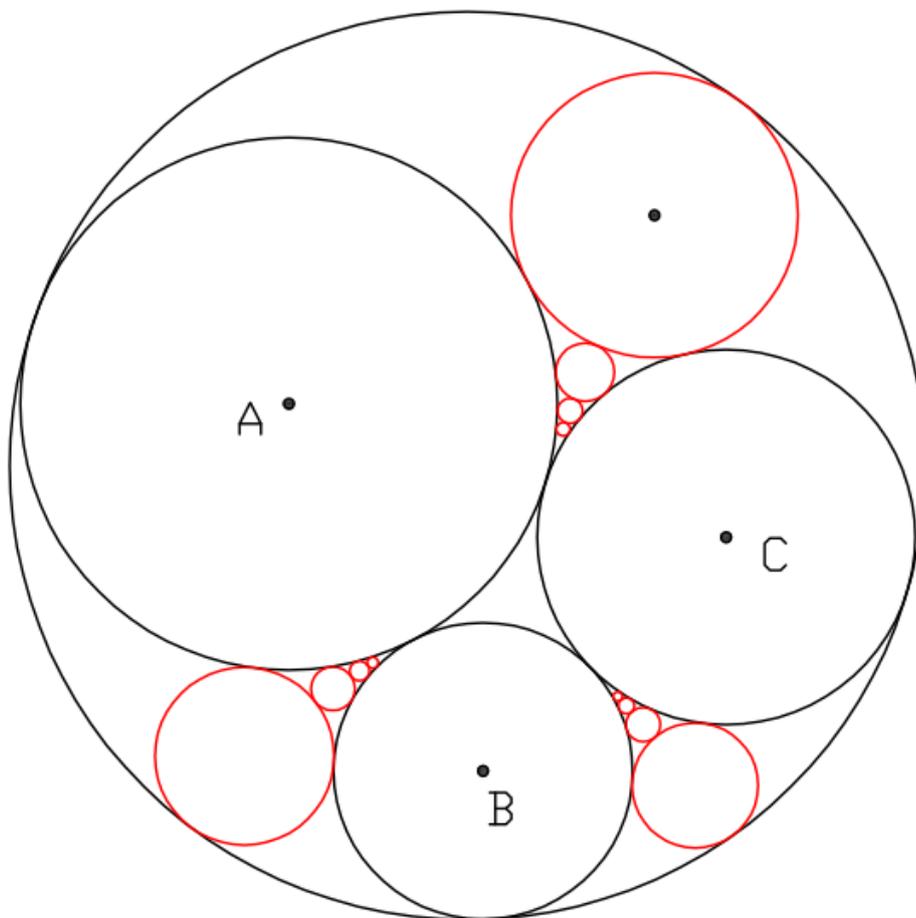


Figure 1

Problem 1. Three lines A_jA_k , B_jB_k , C_jC_k are concurrent for any $j \neq k, j, k = 1, 2, \dots, n$ (Figure 2) .

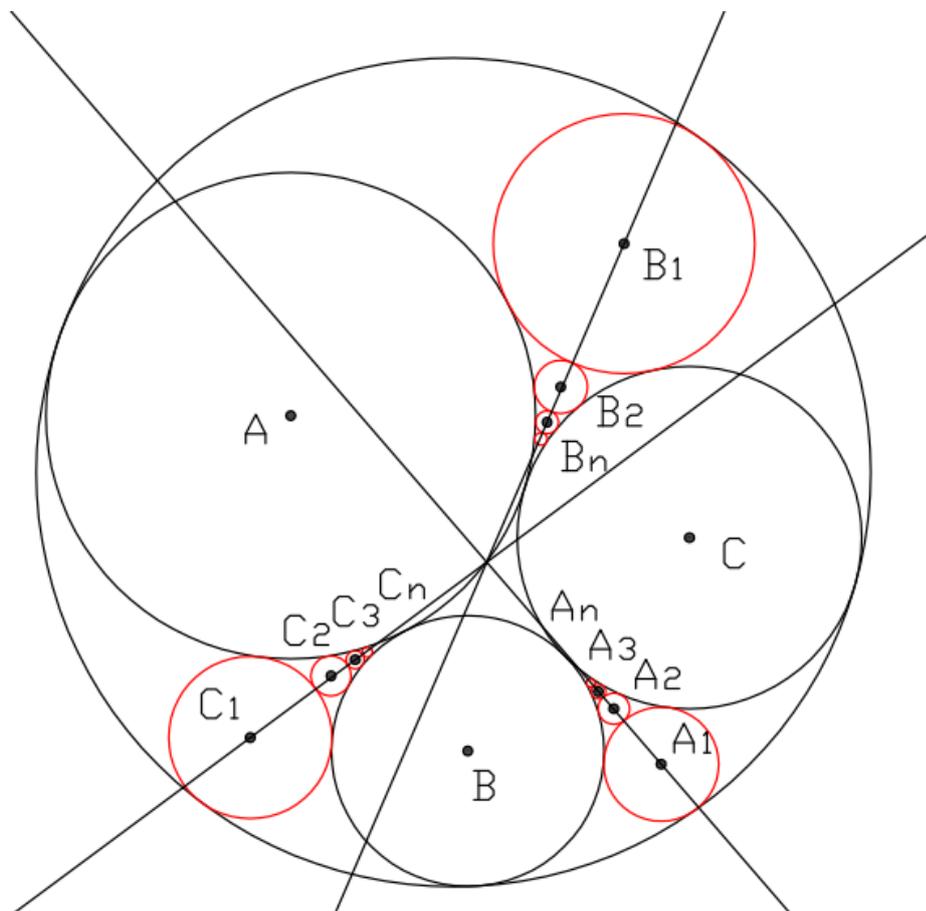


Figure 2

Problem 2. Three line AA_k, BB_k, CC_k are concurrent, for $k = 1, 2, \dots, n$

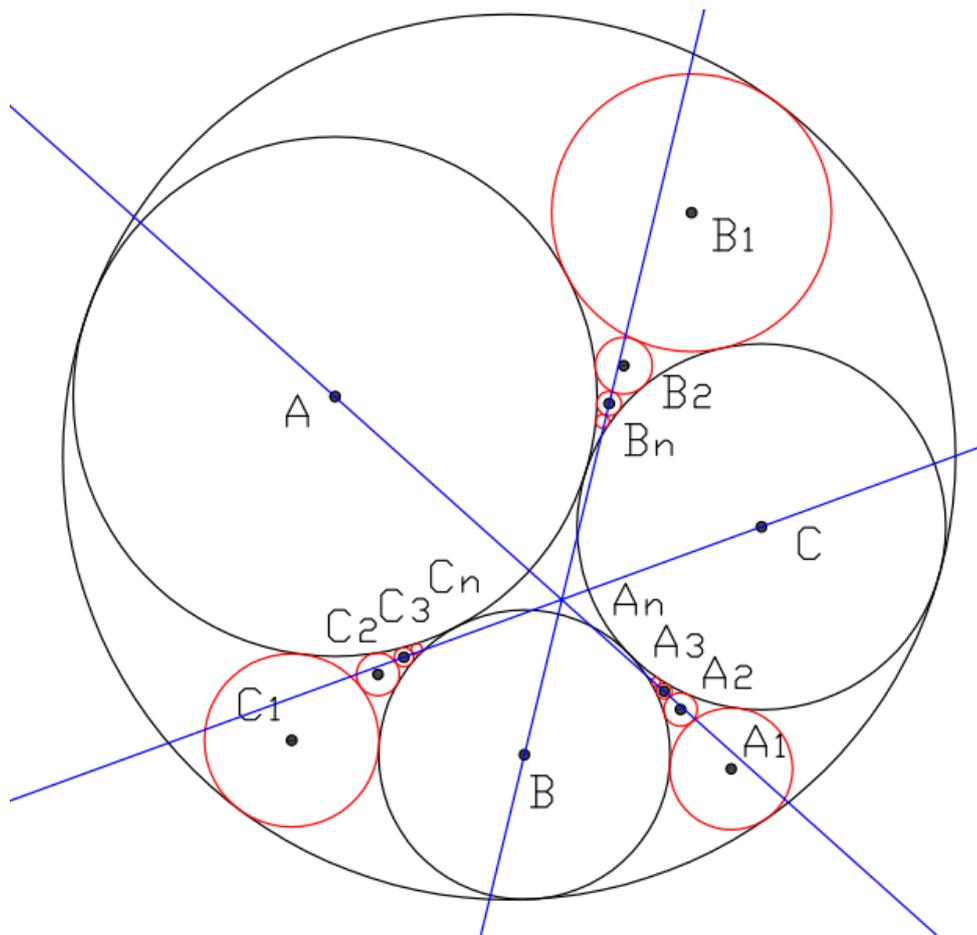


Figure 3

Problem 3. Let (A) tangent to $(B_k), (C_k)$ at A_{ck}, A_{bk} . Define $B_{ck}, B_{ak}, C_{ak}, C_{bk}$ cyclicly. Then six points $A_{bk}, A_{ck}, B_{ck}, B_{ak}, C_{ak}, C_{bk}$ lie on a circle for $k = 1, 2, \dots, n$. and the centers of these new circles lie on a line.

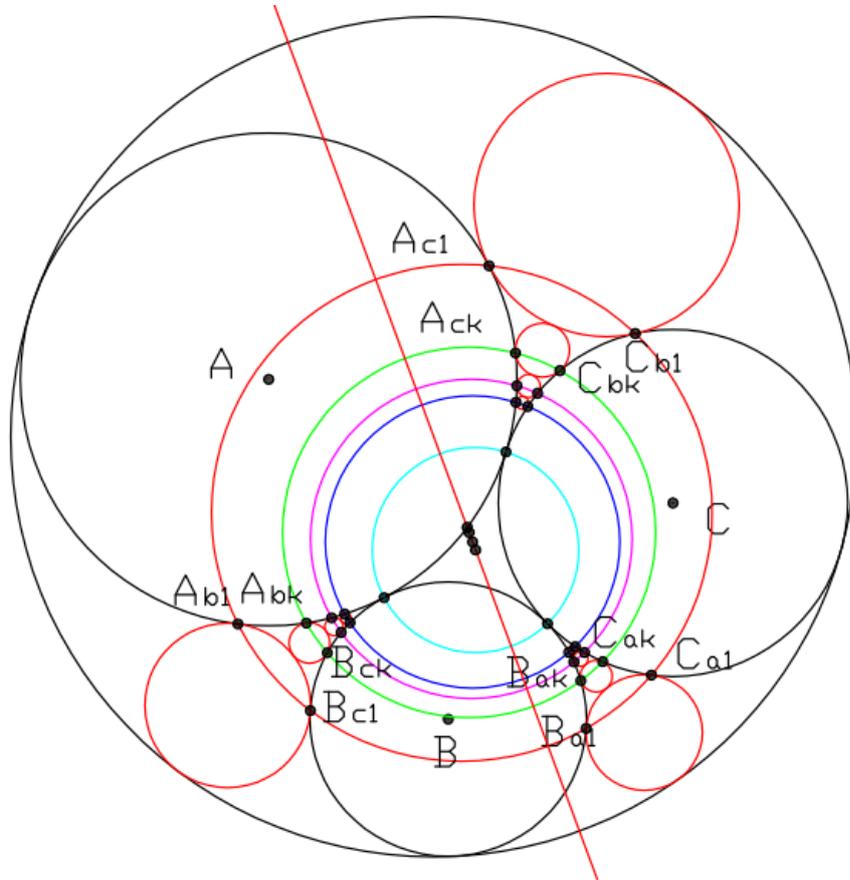


Figure 4

Problem 4. Let (A_k) tangent to (A_{k+1}) at T_{ak} , define T_{ak}, T_{ck} cyclically. Then three lines $T_{aj}T_{ak}, T_{bj}T_{bk}, T_{cj}T_{ck}$ are concurrent for any $j \neq k, j, k = 1, 2, \dots, n$.

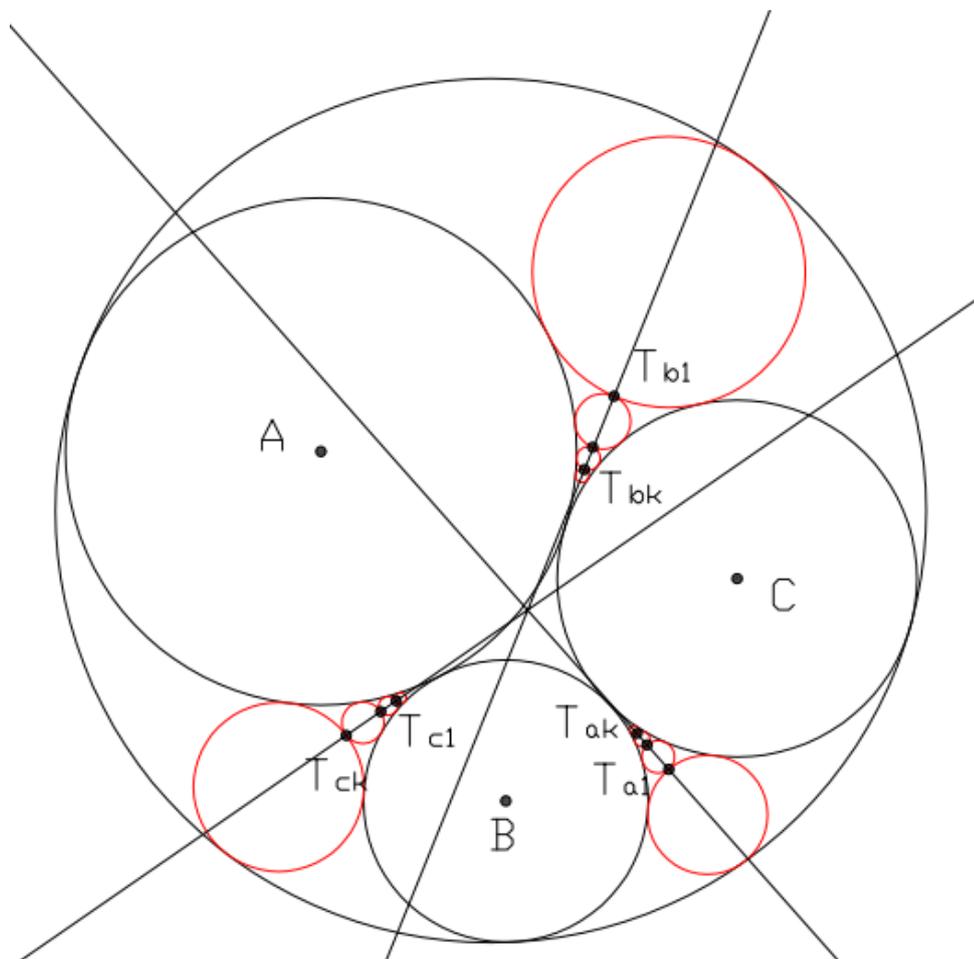


Figure 5

Problem 5. Circle $(T_{ak}T_{bk}T_{ck})$ tangent to six circles $(A_k), (A_{k-1}), (B_k), (B_{k-1}), (C_k), (C_{k-1})$ any $k = 1, 2, \dots, n$.

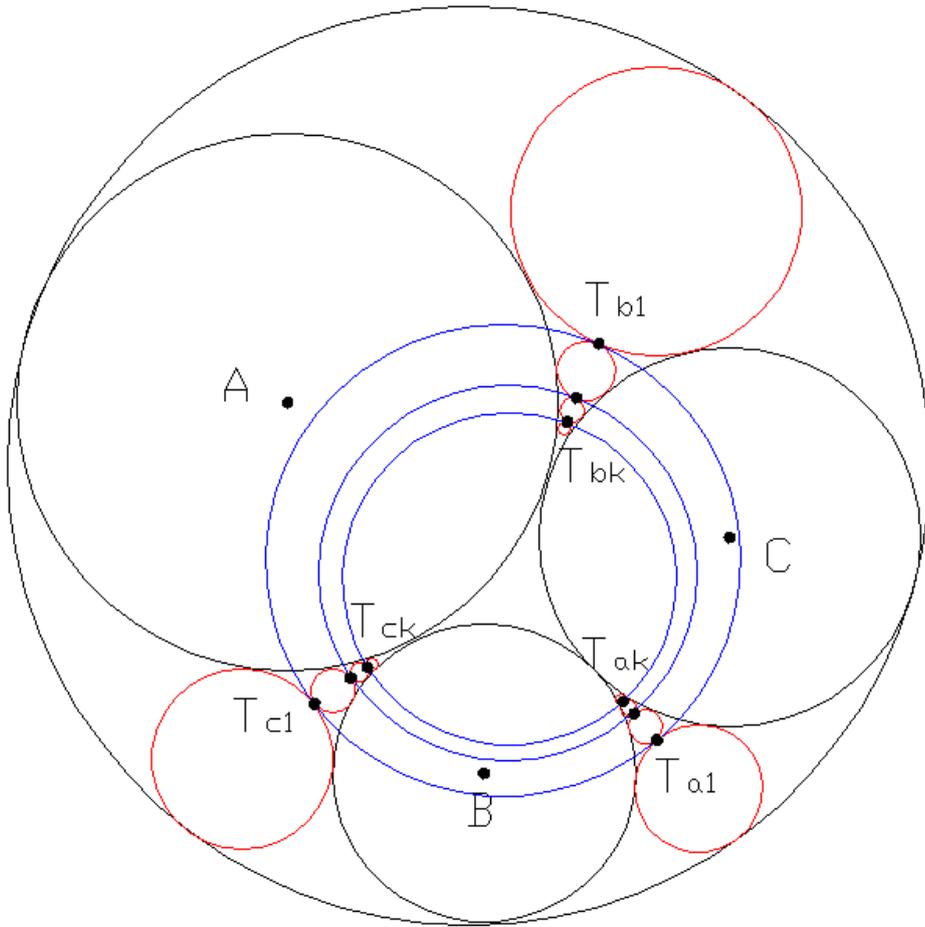


Figure 6

Problem 6. Three lines AT_{ak} , BT_{bk} , CT_{ck} are concurrent, for any $k = 1, 2, \dots, n$.

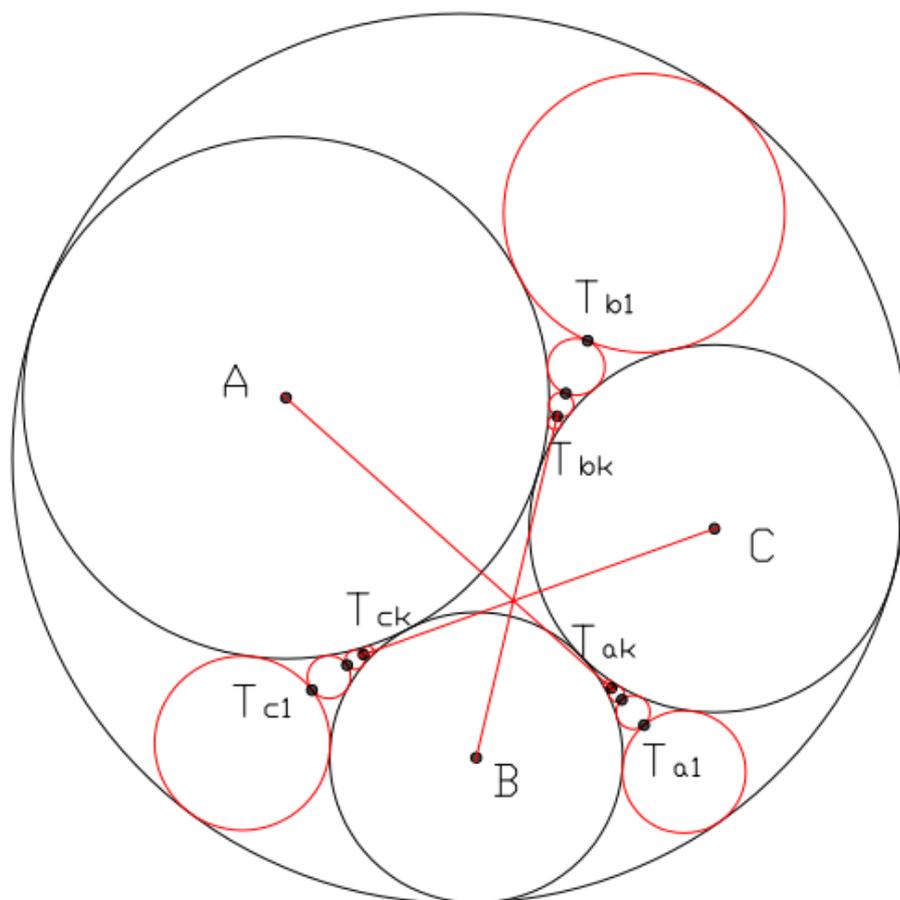


Figure 7

Problem 7. Three lines A_jT_{ak} , B_jT_{bk} , C_jT_{ck} are concurrent for any $j, k = 1, 2, \dots, n$.

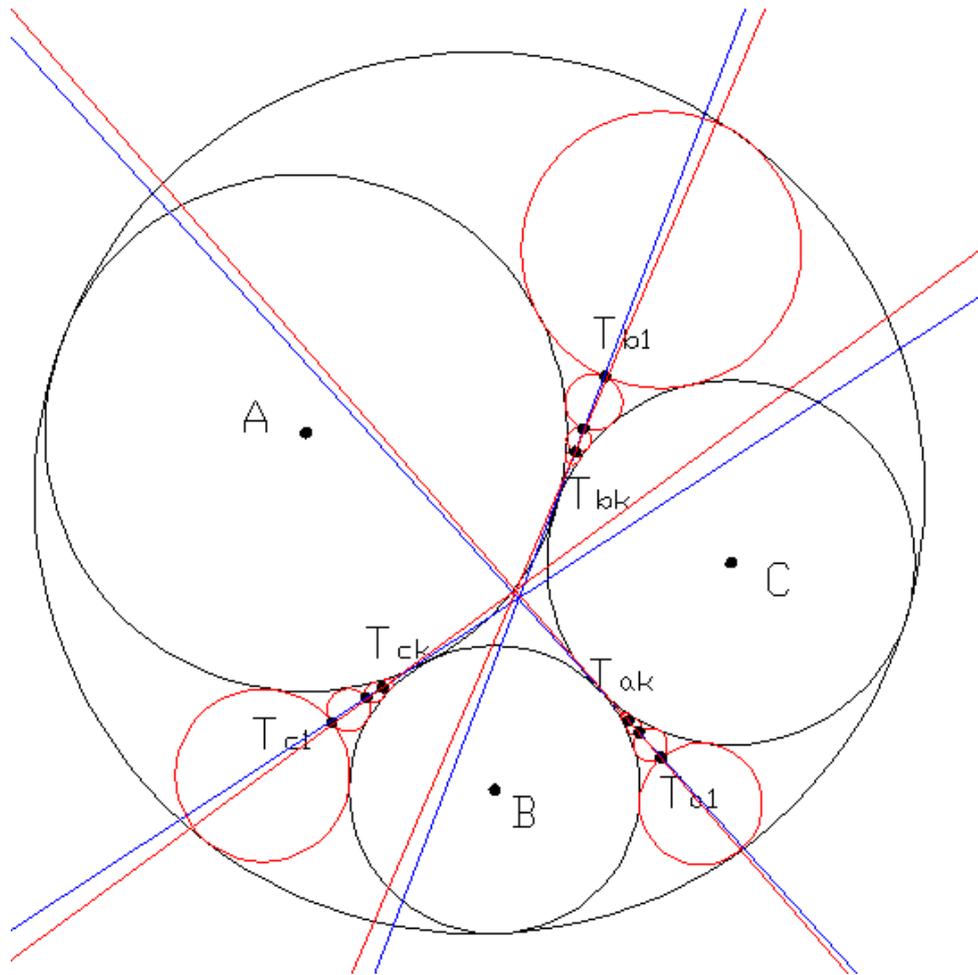


Figure 8

References

- [1] Apollonian gasket, https://en.wikipedia.org/wiki/Apollonian_gasket
- [2] <https://artofproblemsolving.com/community/c6h555078p3225247>

Dao Thanh Oai: Kien Xuong, Thai Binh, Viet Nam

E-mail address: daothanhoai@hotmail.com