Ta chứng minh quy nạp theo n. Thật vậy khi $n = 1$ thì

$$W\left(a_1\right) = \int_0^1 y_1^{a_1} dy = \left.\frac{y_1^{a_1+1}}{a_1+1}\right|_0^1 = \frac{1}{a_1+1} = \frac{\Gamma\left(a_1+1\right)}{\left(a_1+1\right)\Gamma\left(a_1+1\right)} = \frac{\Gamma\left(a_1+1\right)}{\Gamma\left(a_1+2\right)}$$

Giả sử định lý đúng với $n = k-1$, tức là

$$W\left(a_1, a_2, \ldots, a_{k-1}\right) = \frac{\Gamma\left(a_{k-1}+1\right)\ldots\Gamma\left(a_1+1\right)}{\Gamma\left(a_1+a_2+\ldots+a_{k-1}+k\right)}$$

Với $n = k$ ta có

$$W\left(a_1, a_2, \ldots, a_k\right) = \int\cdots\int_{\sum\limits_{i=1}^{k} y_i \leqslant 1, y_j \geqslant 0, j=\overline{1,k}} y_k^{a_k} y_{k-1}^{a_{k-1}} \ldots y_1^{a_1} dy_1 dy_2 \ldots dy_k$$

$$W\left(a_1, a_2, \ldots, a_k\right) = \int_0^1 y_k^{a_k}\left[\int\cdots\int_{\sum\limits_{i=1}^{k-1} y_i \leqslant 1, y_j \geqslant 0, j=\overline{1,k-1}} y_{k-1}^{a_{k-1}} \ldots y_1^{a_1} dy_1 dy_2 \ldots dy_{k-1}\right] dy_k \tag{1}$$

Đổi biến số $z_i = \dfrac{y_i}{1-y_k}$ thì $dy_i = \left(1-y_k\right) dz_i, i = \overline{1, k-1}$ khi đó (1) thành

$$\begin{aligned}
W\left(a_1, a_2, \ldots, a_k\right) &= \int_0^1 y_k^{a_k}\left[\int\cdots\int_{\sum\limits_{i=1}^{k-1} z_i \leqslant 1, z_j \geqslant 0, j=\overline{1,k-1}} \left(1-y_k\right)^{a_1+a_2+\ldots+a_{k-1}+k-1} z_1^{a_1} z_2^{a_2} \ldots z_{k-1}^{a_{k-1}} dz_1 dz_2 \ldots dz_{k-1}\right] dy_k \\
&= \int_0^1 y_k^{a_k}\left(1-y_k\right)^{a_1+a_2+\ldots+a_{k-1}+k-1}\left[\int\cdots\int_{\sum\limits_{i=1}^{k-1} z_i \leqslant 1, z_j \geqslant 0, j=\overline{1,k-1}} z_1^{a_1} z_2^{a_2} \ldots z_{k-1}^{a_{k-1}} dz_1 dz_2 \ldots dz_{k-1}\right] dy_k \\
&= \int_0^1 y_k^{a_k}\left(1-y_k\right)^{a_1+a_2+\ldots+a_k+k-1} W\left(a_1, a_2, \ldots, a_{k-1}\right) dy_k \\
&= W\left(a_1, a_2, \ldots, a_{k-1}\right)\int_0^1 y_k^{a_k}\left(1-y_k\right)^{a_1+a_2+\ldots+a_{k-1}+k-1} dy_k \\
&= W\left(a_1, a_2, \ldots, a_{k-1}\right)\beta\left(a_k+1, a_1+a_2+\ldots+a_{k-1}+k\right) \\
&= W\left(a_1, a_2, \ldots, a_{k-1}\right)\frac{\Gamma\left(a_k+1\right)\Gamma\left(a_1+a_2+\ldots+a_{k-1}+k\right)}{\Gamma\left(a_1+a_2+\ldots+a_k+k+1\right)} \\
&= \frac{\Gamma(1)\Gamma(2)\ldots\Gamma\left(a_{k-1}\right)}{\Gamma\left(a_1+a_2+\ldots+a_{k-1}+k\right)}\frac{\Gamma\left(a_k+1\right)\Gamma\left(a_1+a_2+\ldots+a_{k-1}+k\right)}{\Gamma\left(a_1+a_2+\ldots+a_k+k+1\right)} \\
&= \frac{\Gamma(1)\Gamma(2)\ldots\Gamma\left(a_{k-1}\right)\Gamma\left(a_k+1\right)}{\Gamma\left(a_1+a_2+\ldots+a_k+k+1\right)}
\end{aligned}$$

Vậy

$$W\left(a_1, a_2, \ldots, a_n\right) = \frac{\Gamma\left(a_n+1\right)\Gamma\left(a_{n-1}+1\right)\ldots\Gamma\left(a_1+1\right)}{\Gamma\left(a_1+a_2+\ldots+a_n+n+1\right)}$$