B10>

$$Fourier, Series, \text{for}, \mathrm{x}^2, on, [-\pi, \pi]$$

$$\mathrm{x}^2 \approx \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(\mathrm{nx}) + \sum_{n=1}^{\infty} b_n \sin(\mathrm{nx})$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 dx = \frac{2}{\pi}\int_0^{\pi} x^2 dx = \frac{2}{\pi}\frac{x^3}{3}\Big|_0^{\pi} = \frac{2\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos(nx)dx = \frac{2}{\pi}\int_0^{\pi} x^2 \cos(nx)dx$$

$$\int_0^{\pi} x^2 \cos(nx)dx = \frac{x^2}{n}\sin(nx) + \frac{2x}{n^2}\cos(nx) - \frac{2}{n^3}\sin(nx)$$

$$a_n = \frac{2}{\pi}\left(\frac{x^2}{n}\sin(nx) + \frac{2x}{n^2}\cos(nx) - \frac{2}{n^3}\sin(nx)\right)_0^{\pi} = (-1)^n \frac{4}{n^2}$$

$$b_n = 0$$

$$So:$$

$$\mathrm{x}^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \frac{4}{n^2}\cos(nx), \mathrm{x} \in [-\pi, \pi]$$

$$Set, x = \pi$$

$$--> \pi^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2} --> \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

B11>

$Find:$

$$\sum_{n=1}^{\infty}\frac{\cos n}{n^2}$$

$Lemma:$

$$\sum_{n=1}^{\infty}\frac{\sin(nx)}{n}=\frac{\pi-x}{2},x\in(0,2\pi)$$

$\Pr ove:$

$$Set: g(x)=\frac{\pi-x}{2},x\in(0,2\pi)$$

$Fourier,\exp ansion:$

$$a_0=\frac{1}{\pi}\int_0^{2\pi}\frac{\pi-x}{2}dx=\frac{1}{\pi}\frac{-(\pi-x)^2}{4}\Bigg|_0^{2\pi}=0$$

$$a_n=\frac{1}{\pi}\int_0^{2\pi}\frac{\pi-x}{2}\cos(nx)dx=\frac{1}{2\pi}\int_0^{2\pi}(\pi-x)\cos(nx)dx=\frac{1}{2\pi}\left((\pi-x)\frac{\sin(nx)}{n}\Bigg|_0^{2\pi}+\int_0^{2\pi}\frac{\sin(nx)}{n}dx\right)=$$

$$=\frac{1}{2\pi}\left(\frac{-\cos(nx)}{n^2}\Bigg|_0^{2\pi}\right)=0$$

$$b_n=\frac{1}{\pi}\int_0^{2\pi}\frac{\pi-x}{2}\sin(nx)dx=\frac{1}{2\pi}\int_0^{2\pi}(\pi-x)\sin(nx)dx=\frac{1}{2\pi}\left((\pi-x)\frac{-\cos(nx)}{n}\Bigg|_0^{2\pi}-\int_0^{2\pi}\frac{\cos(nx)}{n}dx\right)=$$

$$=\frac{1}{2\pi}\left(\frac{\pi}{n}-\frac{(-\pi)}{n}-\frac{\sin(nx)}{n^2}\Bigg|_0^{2\pi}\right)=\frac{1}{n}$$

$$\sum_{n=1}^{\infty}\frac{\cos(nx)}{n^2}=\sum_{n=1}^{\infty}\left(\frac{\cos(nx)}{n^2}-\frac{\cos(n.0)}{n^2}+\frac{1}{n^2}\right)=\sum_{n=1}^{\infty}\frac{\cos(ny)}{n^2}\Bigg|_{y=0}^{y=x}+\sum_{n=1}^{\infty}\frac{1}{n^2}=\sum_{n=1}^{\infty}\int_0^x\frac{-\sin(ny)}{n}dy+\zeta(2)=$$

$$=\int_x^0\sum_{n=1}^{\infty}\frac{\sin(ny)}{n}dy+\zeta(2)=\int_x^0\frac{\pi-y}{2}dy+\zeta(2)=\frac{-(\pi-y)^2}{4}\Bigg|_x^0+\zeta(2)=\frac{-1}{4}(\pi^2-(\pi-x)^2)+\zeta(2)$$

$Take, x=1:$

$$\sum_{n=1}^{\infty}\frac{\cos(n)}{n^2}=\frac{-1}{4}(\pi^2-(\pi-1)^2)+\zeta(2)=\frac{\pi^2}{6}-\frac{\pi}{2}+\frac{1}{4}$$