Hanoi Mathematical Society

Hanoi Open Mathematical Olympiad 2010

Senior Section

Sunday, 28 March 2010 08h45-11h45

Important:

Answer all 10 questions.

Enter your answers on the answer sheet provided.

For the multiple choice questions, enter only the letters (A, B, C, D or E) corresponding to the correct answers in the answer sheet.

No calculators are allowed.

Q1. The number of integers $\boldsymbol{n} \in [\mathbf{2000}, \mathbf{2010}]$ such that $\mathbf{2}^{\mathbf{2}\boldsymbol{n}} + \mathbf{2}^{\boldsymbol{n}} + \mathbf{5}$ is divisible by 7, is

(A): 0; (B): 1; (C): 2; (D): 3; (E) None of the above.

Q2. The last 5 digits of the number $\mathbf{5}^{\mathbf{2010}}$ are

(A): 65625; (B): 45625; (C): 25625; (D): 15625; (E) None of the above.

Q3. How many real numbers $\boldsymbol{a} \in (\mathbf{1}, \mathbf{9})$ such that the corresponding number $\boldsymbol{a} - \dfrac{\mathbf{1}}{\boldsymbol{a}}$ is an integer.

(A): 0; (B): 1; (C): 8; (D): 9; (E) None of the above.

Q4. Each box in a $\mathbf{2} \times \mathbf{2}$ table can be colored black or white. How many different colorings of the table are there?

Q5. Determine all positive integer $\boldsymbol{a}$ such that the equation

$$\boldsymbol{2x^2 - 210x + a = 0}$$

has two prime roots, i.e. both roots are prime numbers.

Q6. Let $\boldsymbol{a, b}$ be the roots of the equation $\boldsymbol{x^2 - px + q = 0}$ and let $\boldsymbol{c, d}$ be the roots of the equation $\boldsymbol{x^2 - rx + s = 0}$, where $\boldsymbol{p, q, r, s}$ are some positive real numbers. Suppose that

$$\boldsymbol{M = \frac{2(abc + bcd + cda + dab)}{p^2 + q^2 + r^2 + s^2}}$$

is an integer. Determine $\boldsymbol{a, b, c, d}$.

Q7. Let $\boldsymbol{P}$ be the common point of 3 internal bisectors of a given $\boldsymbol{ABC.}$ The line passing through $\boldsymbol{P}$ and perpendicular to $\boldsymbol{CP}$ intersects $\boldsymbol{AC}$ and $\boldsymbol{BC}$ at $\boldsymbol{M}$ and $\boldsymbol{N}$, respectively. If $\boldsymbol{AP = 3}$cm, $\boldsymbol{BP = 4}$cm, compute the value of $\boldsymbol{\dfrac{AM}{BN}}$?

Q8. If $\boldsymbol{n}$ and $\boldsymbol{n^3 + 2n^2 + 2n + 4}$ are both perfect squares, find $\boldsymbol{n}$?

Q9. Let $\boldsymbol{x, y}$ be the positive integers such that $\boldsymbol{3x^2 + x = 4y^2 + y.}$ Prove that $\boldsymbol{x - y}$ is a perfect integer.

Q10. Find the maximum value of

$$\boldsymbol{M = \frac{x}{2x + y} + \frac{y}{2y + z} + \frac{z}{2z + x}, \;\; x, y, z > 0.}$$
